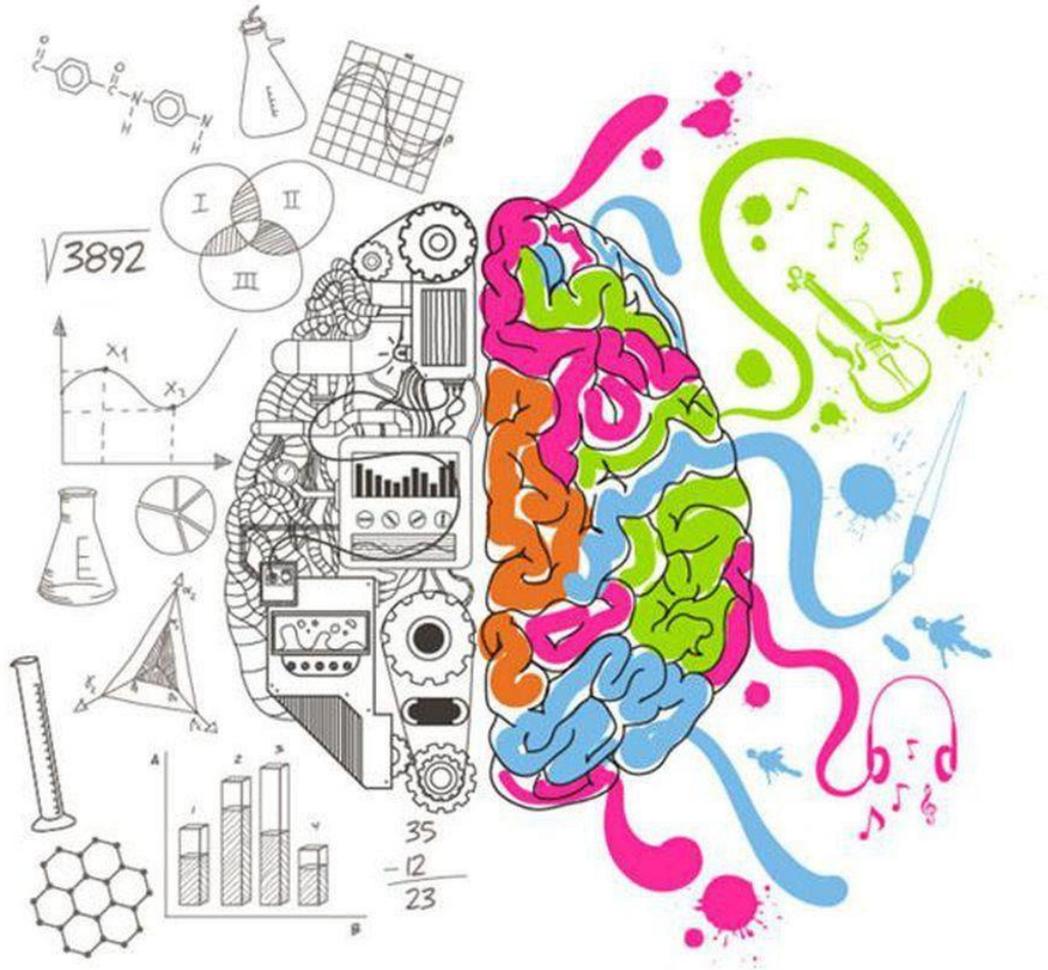




## The Musical Shape of Math

By Michelle Hardy



Right Brain Left Brain function Illustration. FREEPIK.COM

<https://www.forbes.com/sites/autumnadeigbo/2017/03/04/reconcile-your-right-and-left-brain-to-become-a-better-entrepreneur/?sh=62c381367df7>

Individuals who are passionate about music are often considered to be right brain DOMINANT, which means they use more of the right hemisphere of their brain, tending to be more creative or artistic in function. On the other hand, someone who is more ANALYTICAL, logical, or mathematical is said to be left-brained. The reality is that this theory of hemisphere dominance does not hold true. So- not to worry- if you love music, you are not destined to struggle with math! And vice versa- you can be great at math and be a musical PRODIGY (someone with

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exceptional skills or abilities). When doing anything logical *or* creative, we receive information from both sides of the brain. And that is what we hope to do in order to build better brain CONNECTIVITY and overall functioning. Logic and creativity *can and should* exist together for all that we do!

Spell: ARTISTIC                      PASSIONATE                      HEMISPHERE

A prodigy is someone who exhibits \_\_\_\_\_ skills or abilities. EXCEPTIONAL

What side of the brain has been theorized as being more analytical or mathematical? LEFT-BRAIN

OR    What is a characteristic of someone who is considered left-brained?

LOGICAL, ANALYTICAL, MATHEMATICAL

What side of the brain has been theorized as being more creative or artistic?

RIGHT-BRAIN

OR    What is a characteristic of someone who is considered right-brained?

CREATIVE OR ARTISTIC

What is an example of a project that would benefit from the characteristics of both right and left brained individuals?

There are studies that show learning music actually improves academic performance, and there are many connections between math and music. Fundamentally, music is notated in a way that is very reflective of mathematical concepts, such as FRACTIONS. (Fractions are a representation of a part of a whole.) A musical composition is broken down into components called measures. Measures are a unit of time that contain a specific number of beats. The specific number of beats in each measure is then determined by the TIME SIGNATURE.

Spell: PERFORMANCE                      REFLECTIVE                      COMPOSITION

Studies have shown that learning music actually improves \_\_\_\_\_ performance.

ACADEMIC

What concept of math is represented by the time signature? FRACTIONS

Fractions are a \_\_\_\_\_ of a part of a whole. REPRESENTATION

What is something a composition is broken down into? MEASURES, BEATS, TIME

What is a food that the whole is often divided into parts before serving (i.e. fractions)? PIZZA, PIE, APPLE, CAKE

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The time signature is a musical sign written as a fraction that indicates the meter of the song. The NUMERATOR (or top number of the fraction) tells you how many units per measure and the DENOMINATOR (or bottom number of the fraction) tells you what note equals one beat. (One memory trick to remember which is the denominator of the fraction is that denominator and down both start with d! So, the denominator is the number down under!!) Each note is held for various lengths of time; for example, a quarter note receives 1 beat, a half note receives 2 beats, and a whole note receives 4 beats. These measures of notes then rely on patterns, just like math, and the similarities don't stop there!

Spell: METER

MEMORY

PATTERNS

The top number of a fraction is called the \_\_\_\_\_. NUMERATOR

The bottom number of a fraction is called the \_\_\_\_\_. DENOMINATOR

OR Name one part of the fraction. NUMERATOR OR DENOMINATOR

The numerator in the time \_\_\_\_\_ tells you how many units are in the measure. SIGNATURE

OR What is something the time signature tells you?

THE METER OF THE SONG, THE UNITS PER MEASURE, THE UNIT OF TIME THAT EQUALS ONE BEAT

Name a note that might be included in a musical composition. QUARTER NOTE, HALF NOTE, OR WHOLE NOTE

A quarter note receives \_\_\_\_ beat. 1

A half note receives \_\_\_\_ beats. 2

A whole note receives \_\_\_\_ beats. 4

If the time signature of music is 4/4, and a measure has one quarter note already shown, how many beats can be added in the measure? 3

Identify the fraction that is still to be completed in the above example. 3/4

OR Identify the fraction needed to complete the measure.

The numerator is \_\_\_\_\_. 3

The denominator is \_\_\_\_\_. 4

VAKT: Pat your hands on your legs with your speller, or pat on the table with them then shift in meter when the time signature changes. Count 4 measures in 4/4 time (ONE-2-3-4, TWO-2-3-4, THREE-2-3-4, FOUR-2-3-4) then immediately shift into 4 measures of ¾ time (ONE-2-3, TWO-2-3, THREE-2-3, FOUR-2-3). You

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will feel the shift in meter as the tempo remains the same. Shift back and forth between the time signatures to feel the pulse of the music.

In your opinion, are there other similarities that exist between music and math?

Many of the ancient philosophers considered music a branch of mathematics, less because of the involvement of numbers and more because of the geometry and SYMMETRY of music. PYTHAGORAS, an ancient philosopher and mathematician, most known for his geometric equation,  $a^2 + b^2 = c^2$ , discovered the concept of mathematical PROPORTIONS when playing music. When he played his LYRE (a small stringed harp-like instrument of ancient Greece), the sounds he produced from the vibrations of the string depended on the length of the string; and those strings were mathematically proportional to each other. (Proportional means corresponding in size or amount to something else.) He was interested in understanding the relationships between various pitches and the relative spacing between the notes.

Spell: GEOMETRY            RELATIVE            VIBRATIONS

Pythagoras was an ancient \_\_\_\_\_ and mathematician. PHILOSOPHER

OR Pythagoras was a \_\_\_\_\_. MATHEMATICIAN, PHILOSOPHER

\_\_\_\_\_ means corresponding in size or amount to something else.

PROPORTIONAL

What is one similarity between music and math? NUMBERS, SYMMETRY, GEOMETRY

What is the name of the geometric equation,  $a^2 + b^2 = c^2$ ? THE PYTHAGOREAN THEORUM

Homophones are words that are pronounced the same but have different meanings, origins, or spelling. What does the homophone for the stringed instrument, the lyre, mean? (LIAR) SOMEONE WHO DOES NOT TELL THE TRUTH

The Pythagorean theorem ( $a^2 + b^2 = c^2$ ) is a fundamental geometric relationship among the three sides of a right triangle. Solve for x.

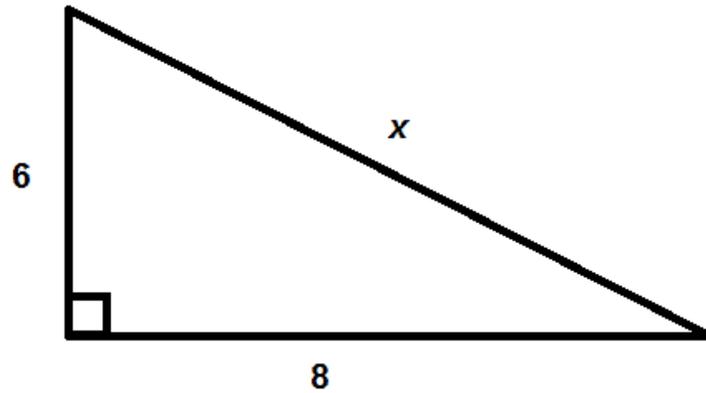
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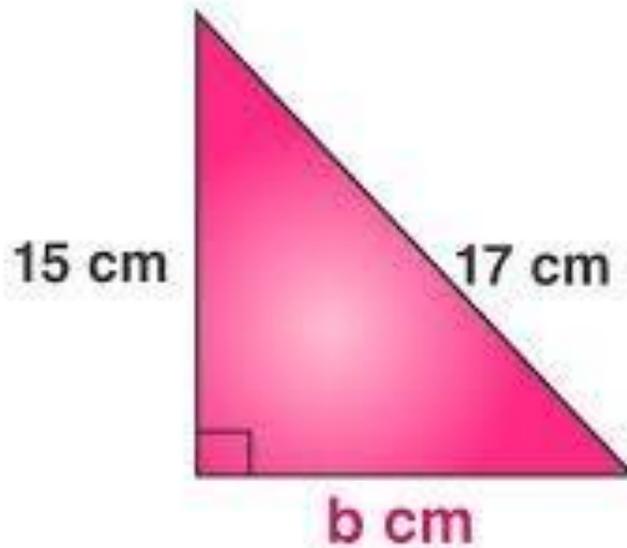
$$6^2 + 8^2 = x^2$$

$$6 \times 6 = \underline{\quad} 36$$

$$8 \times 8 = \underline{\quad} 64$$

$$36 + 64 = \underline{\quad} 100$$

$$100 = x^2; \text{ therefore, } x = \underline{\quad} 10$$



Solve for b in the following equation.

$$15^2 + b^2 = 17^2$$

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$$15 \times 15 = \underline{\quad} 225$$

$$17 \times 17 = \underline{\quad} 289$$

$$225 + b^2 = 289$$

$$b^2 = 289 - 225$$

$$289 - 225 = \underline{\quad} 64$$

The square root of 64 = 8 cm

VAKT: Using the geometry problem above, complete the following actions.

- Point to the longest side of the triangle.
- Trace your finger along the side that measures 15 cm.
- Touch the right angle.



A lyre with varied string lengths  
<https://luthieros.com/lyre2project/>

Pythagoras observed that if two strings were exactly the same length, tension, and thickness, the plucked strings created the same pitch (or tone) and sounded good together. CONSONANCE is the musical term for this pleasing sounding harmony. Sometimes, he found that certain pitches when played together did *not* sound harmonious, and this is called DISSONANCE in music. All of the spaces between the notes, whether consonant or dissonant, are called INTERVALS, and the movement between the two is what creates shape and direction in the music itself as it increases and decreases harmonic tension.

Spell: PLUCKED

LYRE

HARMONY

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What do you call tones whose intervals sound good together?

CONSONANT/CONSONANCE

What do you call tones that sound less harmonious or create tension when played together? DISSONANT/DISSONANCE

OR What is one word that describes how intervals sound together?

CONSONANCE, DISSONANCE, PLEASING, HARMONIOUS

Pitch or sound is determined by the length, \_\_\_\_\_, and thickness of the string.

TENSION

OR What is something that determines the pitch on a stringed instrument?

LENGTH, TENSION, OR THICKNESS OF A STRING

What does the movement between notes create? INTERVALS, SHAPE OR DIRECTION OF MUSIC, HARMONIC TENSION

Name another stringed instrument besides the lyre. GUITAR, VIOLIN, UKULELE

Pythagoras found that on the lyre, the length of strings was proportional to the others and the musical sound was predictive and consonant. For example, if one string was exactly one-half the length of the other string, the shorter string's pitch was the same tone only higher pitched. This interval is called an OCTAVE (a series of eight notes occupying the interval between two notes). If one string had a length that was two-thirds the length of the other, the sound of the strings played together was identified as a perfect 5<sup>th</sup> interval. Regardless of the size of the instrument, the strings always need to be proportional to each other to create the interval sound we are expecting; therefore, a larger lyre would be mathematically similar to a smaller lyre, with their corresponding strings or segments being proportional to each other. Instruments today do not rely solely on Pythagoras' mathematical findings but instead rely more on different thicknesses and tensions of the strings to produce the consonant or dissonant intervals that make music interesting.

Spell: PREDICTIVE

PROPORTION

SEGMENTS

An \_\_\_\_\_ is a series of eight notes between two notes and produces a consonant sound. OCTAVE

A string of \_\_\_\_\_ length produces a higher pitch. SHORTER

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OR Pitch is changed when a string's length is \_\_\_\_\_. MODIFIED, SHORTER, LONGER

If you have a string 10 inches in length, what length would the additional string have to be to produce an interval an octave higher?

$10/2 = \underline{\quad} 5$  inches

OR 10 divided by  $\underline{\quad} = \underline{\quad}$   $10/2 = 5$

How would you produce an octave lower based on a 10-inch string?

$10 \times 2 = \underline{\quad} 20$  inches (Double the length of the original string for a lower sound)

OR  $10 \times \underline{\quad} = \underline{\quad}$   $10 \times 2 = 20$  inches

If you had a string 21 inches long, and you wanted to create a perfect 5<sup>th</sup> interval, what would the additional string length need to be?

First, you need to multiply by what fraction?  $2/3$

OR What is the numerator of the fraction that you would multiply by? (2)

What is the denominator of the fraction that you would multiply by? (3)

$21 \times 2/3 = \underline{\quad} 14$

OR  $\frac{21}{1} \times \frac{2}{3} = \frac{?}{?}$  The numerator is  $\underline{\quad}$ . 42

The denominator is  $\underline{\quad}$ . 3

$42/3 = \underline{\quad} 14$  The additional string would be 14 inches long.

Applying these math skills outside of music, solve a more practical problem. If you want to create a garden plot that is mathematically similar to your existing 8' x 10' plot, and the smaller side has to be 6 ft in length due to the available space, how long would the longer side have to be?

Step 1: Setting this problem up to have garden plots that are proportional, what size is the 6' proportional to so that we can set up our equation? 8'

What number goes over x in this algebraic equation?

$\frac{8}{6} = \frac{?}{x}$   $? = 10$

$6 \times x$

Step 2: Cross multiply.  $\underline{\quad} \times x = 60$   $8x = 60$

Step 3:  $60/8 = \underline{\quad} .57$

The size of the new plot is 6' by  $\underline{\quad}$ ? 7.5'

While the connections between music and math originated many, many centuries ago, the work continues today in more depth and exploration, and we have only scratched the surface. Dmitri TYMOCZKO, a music professor at PRINCETON

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University, has analyzed music, focusing his work on the translation of music specifically into concepts of geometry. He has mapped out AURAL melodies (aural is related to the sense of hearing) onto geometric graphics (visually). His work demonstrates that while music does not follow the scholarly works of mathematicians in its' process and creation, fascinatingly, the patterns, ratios, and formulas of melody are similar to those observed in geometry. It is all about the arrangement of points and how they relate somehow in shape, space, and distance to each other. The work of ancient philosophers and current day musicians and THEORISTS, takes the musings of Pythagoras to a more solid foundation of investigation and evidence, "There is geometry in the humming of the strings, there is music in the spacing of the spheres."

Spell: CENTURIES                  FORMULAS                  TRANSLATION

Tymoczko mapped out \_\_\_\_\_ melodies onto visual geometric graphics to demonstrate the similarities between music and geometry. AURAL

Pythagoras said, "There is geometry in the \_\_\_\_\_ of the strings..."

HUMMING

"There is music in the spacing of the \_\_\_\_\_." SPHERES

Aural is related to what? SENSE OF HEARING

OR What is one sensory representation Tymoczko used to map the music and math connection? AURAL/AUDITORY, VISUAL

What is one feature of geometry that is observed in melody?

PATTERNS, RATIOS, FORMULAS

Geometry is about the arrangement of points and how they relate somehow in \_\_\_\_\_ to each other. SHAPE, SPACE, DISTANCE

Other than geometry, what is another branch of math? ALGEBRA,

TRIGONOMETRY, CALCULUS, STATISTICS

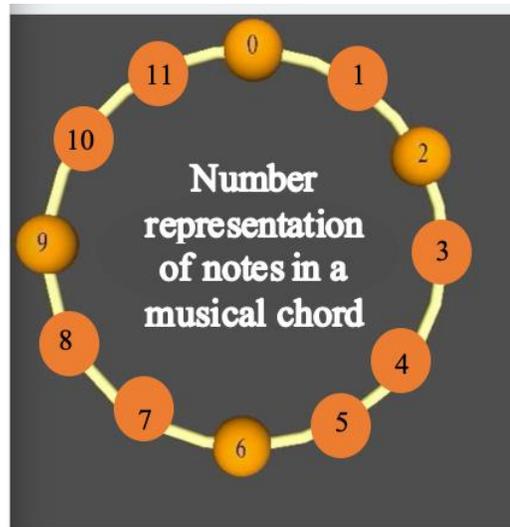
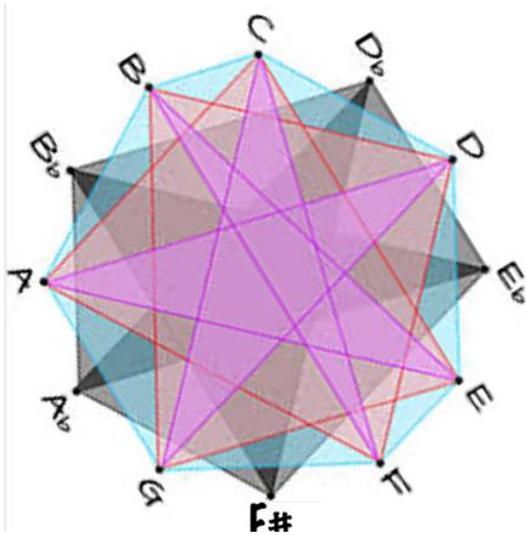
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The graphic above on the left is a geometric shape showing all possible tone connections on the musical scale. The circle in the black box above represents Tymoczko’s graphic representation of a musical chord. The points on each picture correspond and overlap. For example, C corresponds to 0,  $D_b$  (called d flat) corresponds to 1, and so on.

Using these two visuals, create coordinates (matching a number to the letter in the pictures) to correspond with the notes in a C chord: C, E, and G.

(C,     ) 0                      (E,     ) 4                      (G,     ) 7

Plot the points using your coordinates on the graph at the end of the lesson. Then connect the points. (CRP can mark as the speller touches the point on the graph or the graph can be slid into a sheet protector to mark with dry erase markers. The points and line are represented in blue on the answer key.)

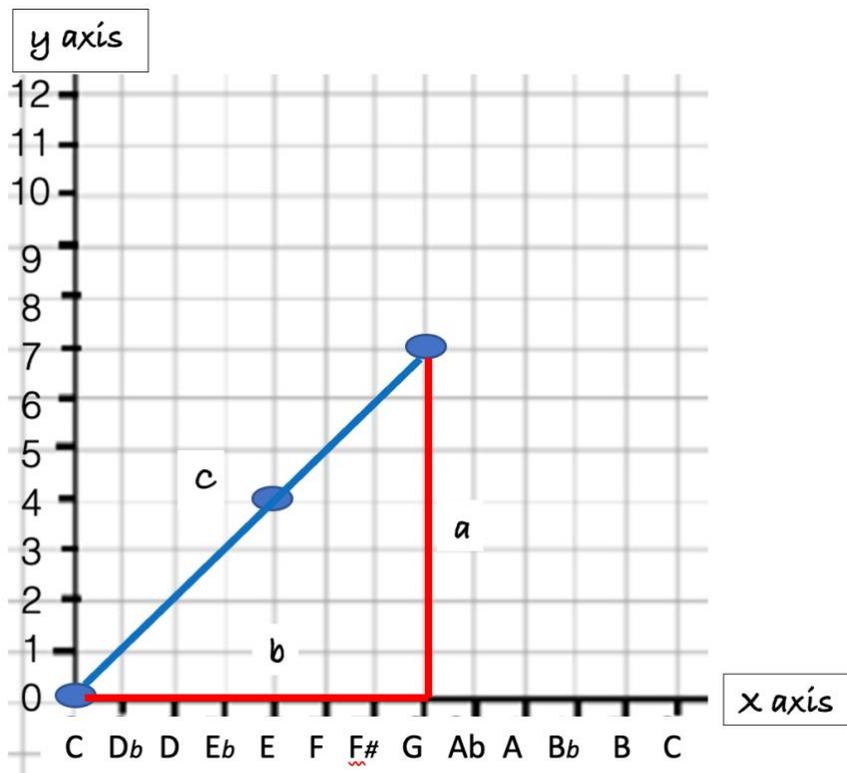
What geometric figure is formed when you connect the points? A LINE

Draw a line down from the (G, 7) coordinate to the x axis. Then draw a line across the x axis to connect the points and form a triangle. (red lines on answer key)

Label the vertical line “a”, the horizontal line “b”, and the diagonal line “c.”

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### ANSWER KEY

What is the length of a? 7

What is the length of b? 7

Given your a and b lengths, what is the length of c?

$$7^2 + 7^2 = \underline{\quad}$$

Step 1  $7 \times 7 = \underline{\quad} 49$

$$49 + 49 = \underline{\quad} 98$$

c = the square root of  $\underline{\quad} 98$

What is the closest perfect square to the answer of  $49 + 49$ ? 100

What is the square root of 100? 10

If  $a=7$  and  $b=7$ , then c is  $\underline{\quad} 10$ . LESS THAN, GREATER THAN, OR EQUAL TO  
(LESS THAN IS THE CORRECT ANSWER)

What is the name of the triangle that has two equal sides like the one in your graph? ISOSCELES TRIANGLE

OR Name another type of triangle. EQUILATERAL TRIANGLE, ISOSCELES TRIANGLE, SCALENE TRIANGLE, ACUTE TRIANGLE, OBTUSE TRIANGLE, RIGHT TRIANGLE

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OR VAKT: If your speller has challenges with ocular motor use this VAKT instead of the above problem with larger targets. Use the visuals at the end of the lesson. The graphic on the left is a geometric shape showing all possible tone connections on the musical scale. The circle in the black box represents Tymoczko's graphic representation of a musical chord. The points on each picture correspond and overlap.

- Point to the number 2 on the right. Touch the letter name (or note) that corresponds to that number on the image on the left. 2 corresponds to D
- Point to the number 6 on the right. Touch the letter name (or note) that corresponds to that number on the image on the left. 6 corresponds to F#
- Point to the number 9 on the right. Touch the letter name (or note) that corresponds to that number on the image on the left. 9 corresponds to A
- Point to the number 0 on the right. Touch the letter name (or note) that corresponds to that number on the image on the left. 0 corresponds to C

Use the paper piano on the last page. If you look at the 12 points on the geometric shape, you can see how they correspond to the piano keyboard in half steps. Each key (black or white) is a half-step away from its next-door neighbor. Touch each half step and determine what is the distance between C and A in half

steps? 9

Watch this video to see how the music of Chopin is depicted on a circular "pitch class space". It shows the movement of points in the melody and the distance and space in between the various tones: <https://vimeo.com/20300784> [0-1:00]

What is a topic that you have only "scratched the surface" of and you would be interested to learn more about?

Creative Writing:

Considering the theory of hemisphere dominance, what traits do you relate to that might give one the perspective that you are right or left brained?

If the world were to exist with only music or only math, what would your preference be and why? Is this possible to even consider?

Pythagoras said, "There is geometry in the humming of the strings, there is music in the spacing of the spheres." Where do you see geometry and music overlap?

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Michelle Hardy is currently a S2C Practitioner in Training who lives in San Diego, CA. She has worked for over 25 years as a Music Therapist rooted in neurodevelopment and has a passion for supporting individuals with autism to find their voice. She loves to laugh (she is the funniest girl she knows), drink coffee, hike, and spend time with her husband and four kids. This lesson brought together her love of autism, music, and math- and she was grateful to have her Mom, her first and favorite math teacher, to fact check her problems as an

unpaid math 'consultant'. 😊

### Extension activities:

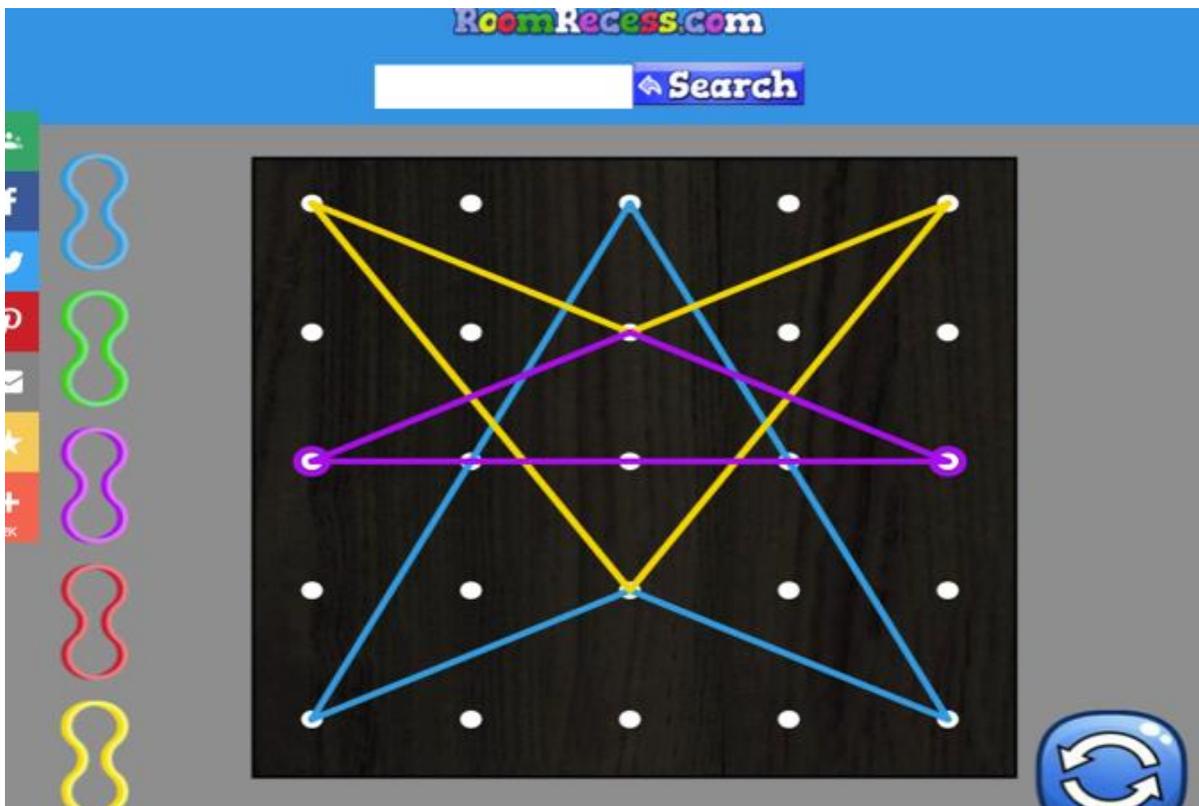
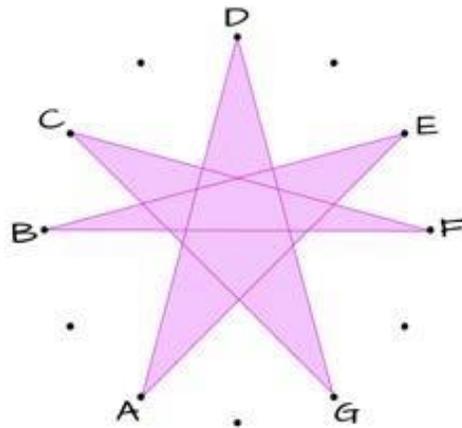
Scroll down on this website and print out a few geometric shapes depicting various intervals and circular displays of musical tones:

<https://roelhollander.eu/en/blog-music/music-geometry/>

Use a geoboard to create the shape with rubber bands. (Example visual)

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Explore the sounds of the piano. Play various notes and try to identify the intervals that make a consonant sound (octaves, fifths, thirds). Look at the distance and the spacing on the circular representation of an octave and see how that distance translates on the circular representation.

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Resources:

<https://www.healthline.com/health/left-brain-vs-right-brain#left-brainright-brain-theory>

<https://www.postcrescent.com/story/life/2017/06/03/how-mathematics-relates-music/359020001/>

[https://www.phys.uconn.edu/~gibson/Notes/Section3\\_2/Sec3\\_2.htm](https://www.phys.uconn.edu/~gibson/Notes/Section3_2/Sec3_2.htm)

[https://en.wikipedia.org/wiki/Dmitri\\_Tymoczko](https://en.wikipedia.org/wiki/Dmitri_Tymoczko)

<http://www.ams.org/publicoutreach/math-and-music>

<https://dmitri.mycpanel.princeton.edu/ChordGeometries.html>

<https://roelhollander.eu/en/blog-music/music-geometry/>

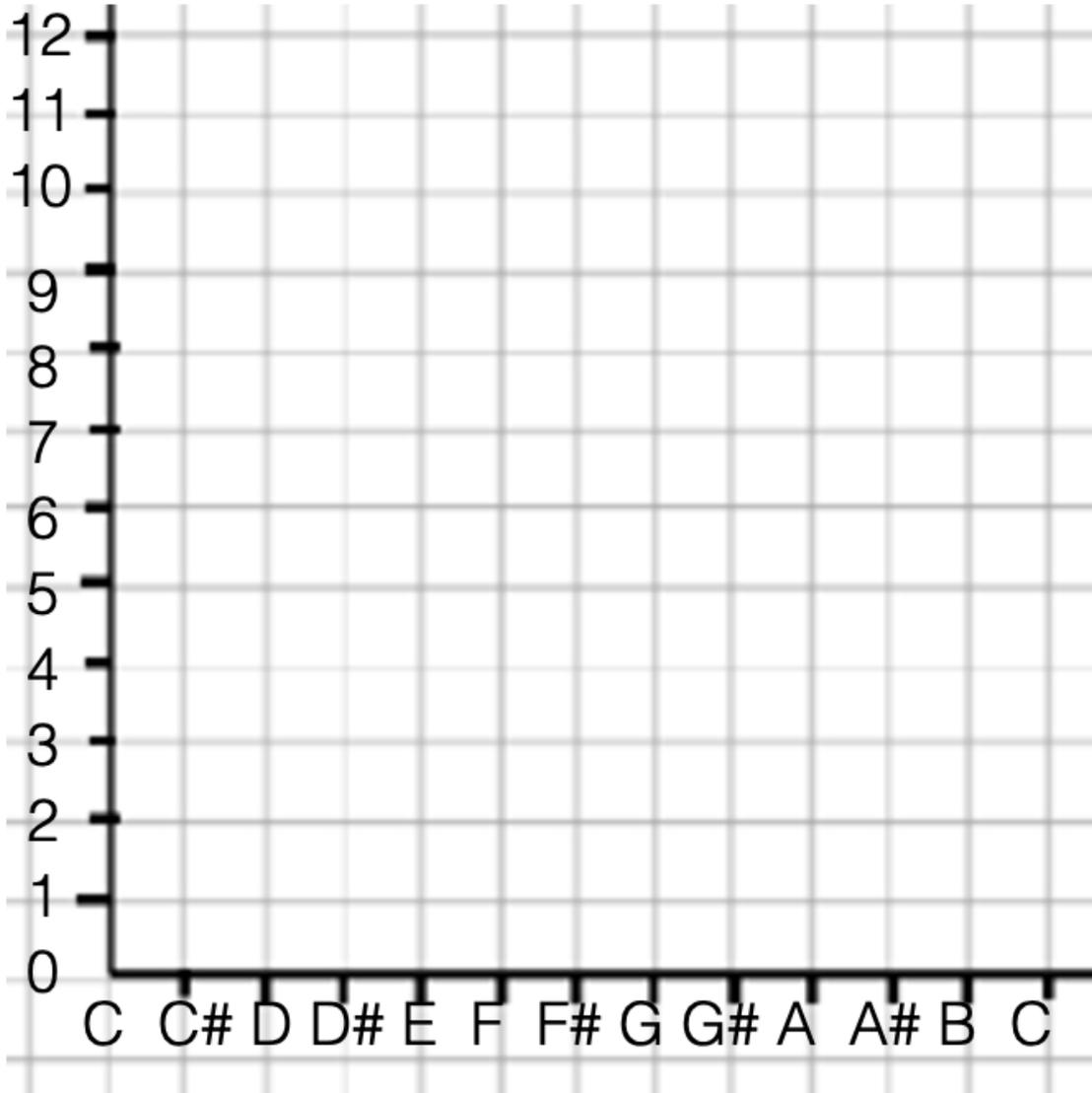
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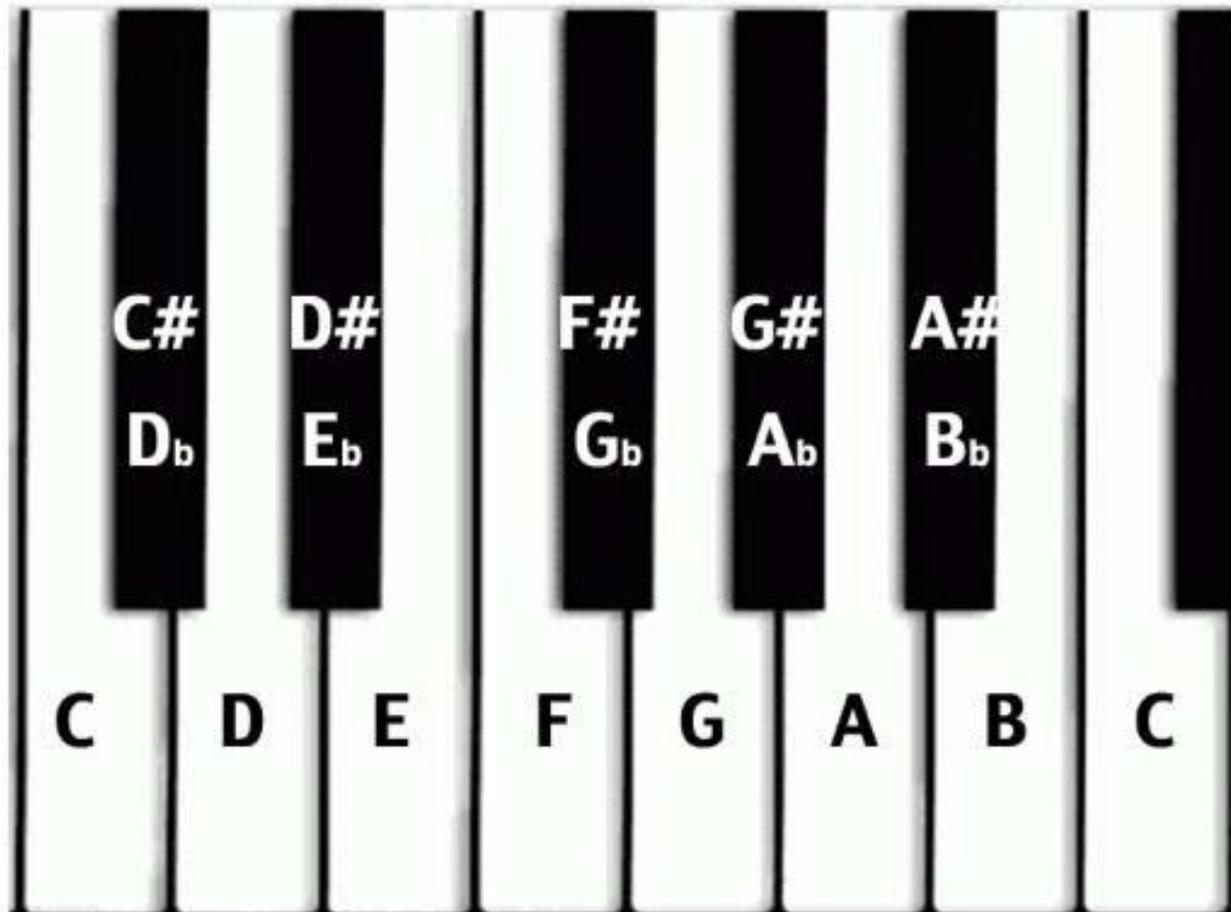
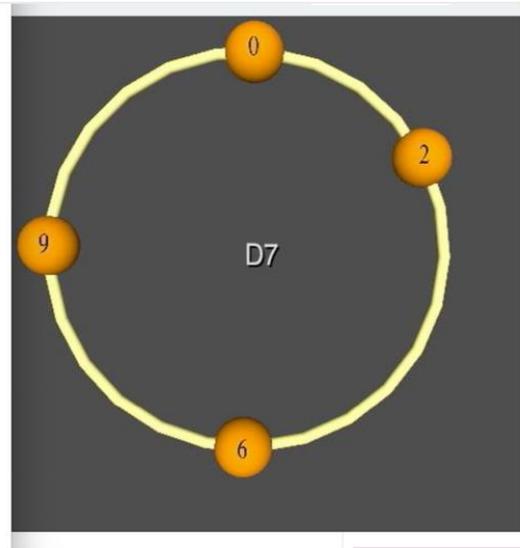
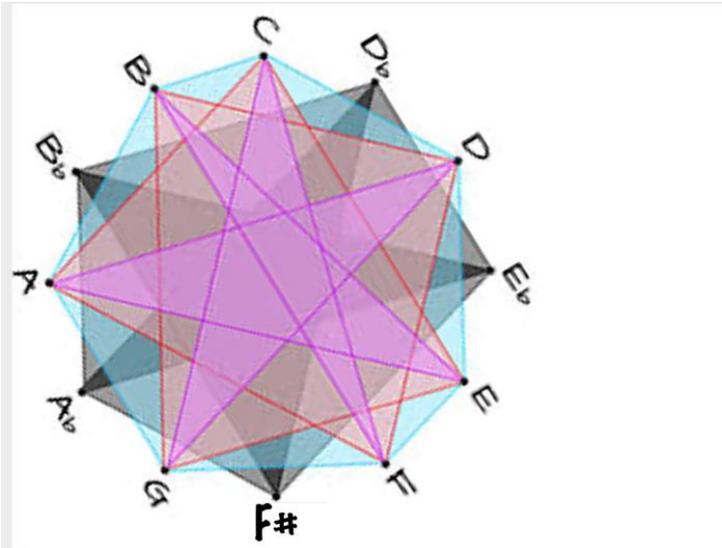
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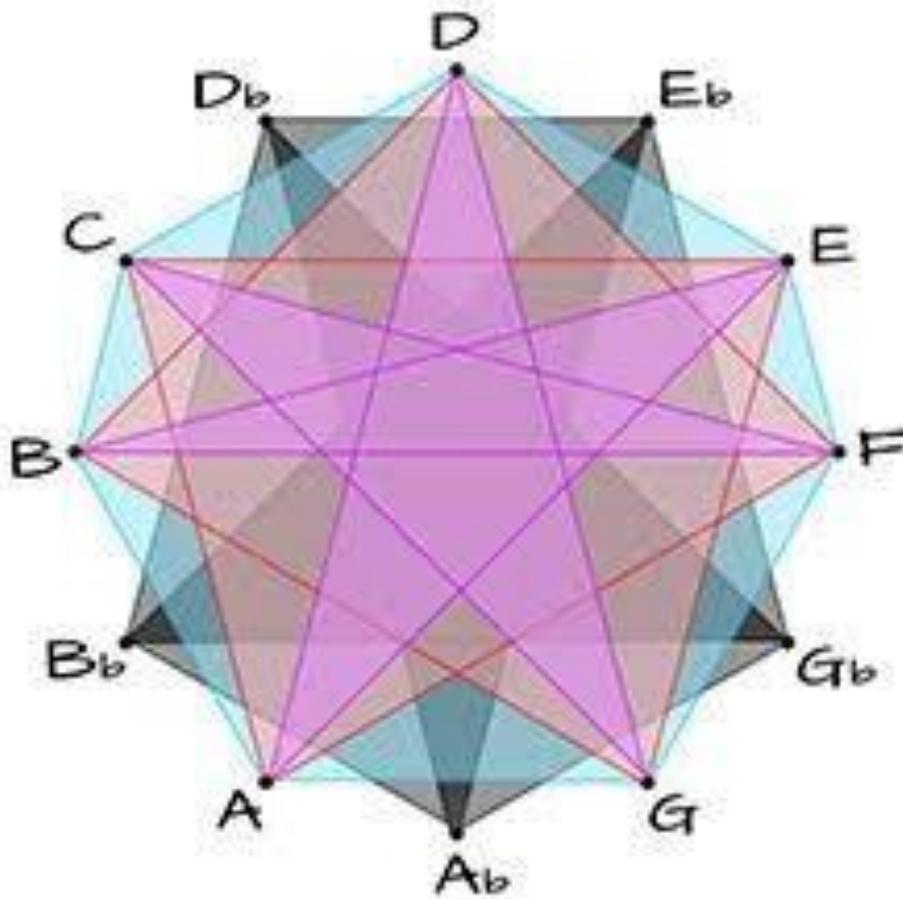
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